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OPTIMAL HEATING SURFACE IN A MULTISTAGE COUNTERCURRENT FLUIDIZED-BED HEAT EXCHANGER

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Equations are derived for calculating the intermediate temperatures of the heating medium such as to give the minimal total surface area in a multistage countercurrent heat exchanger having tubes immersed in a fluidized bed.

Heat exchangers having surfaces immersed in fluidized beds are increasingly being used in industry, on account of the properties of fluidized beds such as high heat-transfer coefficient, isothermal conditions in the bed, and absence of local overheating.

On the other hand, the isothermal condition is a disadvantage, since it restricts the maximum temperature of the cooling medium. The heat-balance equation for a one-stage exchanger is as follows (here and subsequently it is assumed that the heating medium passes within the tubes, while the cooler medium passes through the bed):

$$w(t_1 - t_2) = \theta_1 - \theta_2, \tag{1}$$

which goes with the condition for the temperature difference between the heating fluid and the cooling one:

$$t_2 \geqslant \theta_1, \tag{2}$$

which gives an expression for the temperature of the cooler fluid in a one-stage equipment; in the limiting case, for $t_2 = \theta_1$, the maximum temperature in the cooler medium is

$$\theta_{1\max} = \frac{wt_1 + \theta_2}{1 + w}. \tag{3}$$

One therefore usually employs a multistage system in order to raise the final temperature of the cooler medium.

The gas temperature falls in a single stage of a fluidized-bed heat exchanger, while the temperature of the bed remains constant, and therefore one cannot say one has countercurrent flow or direct flow in a single stage; however, in a multistage system it is possible to distinguish the two types.

In a direct-flow system, the two fluids enter the first stage and pass in series through all stages to emerge from stage n. In the countercurrent case, the two fluids enter at opposite ends of the chain and move in opposite directions.

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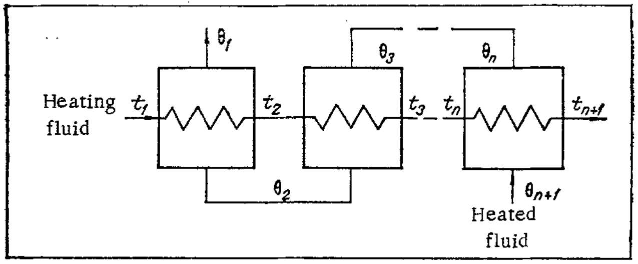

Fig. 1. Scheme for a multistage countercurrent fluidized-bed heat exchanger.

Here we consider a countercurrent system as shown in Fig. 1.

The initial data for the design are w_h, w_c, t_1, θ_1, θ_{n+1}; the calculation has to provide the areas F_i and F of the surfaces. The calculation is based on the heat-balance equation (4) and the heat-transfer equation (5) for each of the individual stages:

$$w_h(t_i - t_{i+1}) = w_c(\theta_i - \theta_{i+1}), \tag{4}$$

$$w_h(t_i - t_{i+1}) = \frac{K_i F_i (t_i - t_{i+1})}{\ln \dfrac{t_i - \theta_i}{t_{i+1} - \theta_i}}. \tag{5}$$

From (5) we get F_i and F:

$$F_i = \frac{w_h}{K_i} \ln \frac{t_i - \theta_i}{t_{i+1} - \theta_i}. \tag{6}$$

The total surface area of all heat exchangers is

$$F = \Sigma F_i = w_h \left(\frac{1}{K_1} \ln \frac{t_1 - \theta_1}{t_2 - \theta_1} + \dots + \frac{1}{K_n} \ln \frac{t_n - \theta_n}{t_{n+1} - \theta_n} \right). \tag{7}$$

The areas in the preceding exchangers (along the path of the hotter medium) decrease as the intermediate temperatures t_2, t_3, ..., t_n increase, while the areas in the later stages increase. There is a turning point in the sum of the areas in this system of fluidized-bed exchangers having intermediate temperatures $\Delta F_i \neq \Delta F_{i+1}$, which can be represented as $F = f(t_2, t_3, \dots, t_n)$, the minimum occurring for w_h, w_c, t_1, t_{n+1}, θ_1, θ_{n+1}, all constant.

To determine the critical points in (7) we take the partial derivative

$$\frac{\partial F}{\partial t_i} = w_h \left[\frac{K_{i-1}(t_{i-1} - \theta_{i+1})(t_i - \theta_{i-1}) - K_i(t_i - \theta_i)(t_{i+1} - \theta_i)}{K_{i-1}K_i(t_i - \theta_i)(t_{i+1} - \theta_i)(t_i - \theta_{i-1})} \right]. \tag{8}$$

The partial derivative of (8) is zero, infinite, or has no value at a critical point. We equate the numerator and denominator in (8), in turn, to zero to get for heat exchanger i that

$$K_{i-1}(t_{i+1} - \theta_{i+1})(t_i - \theta_{i-1}) - K_i(t_i - \theta_i)(t_{i+1} - \theta_i) = 0, \tag{9}$$

$$t_i - \theta_i = 0, \tag{10}$$

$$t_{i+1} - \theta_i = 0, \tag{11}$$

$$t_i - \theta_{i-1} = 0. \tag{12}$$

It is found that (7) has a minimum for the t_i defined by (9).

We have from (4) that

$$wt_i - \theta_i = c_1 - \text{const}. \tag{13}$$

We put

$$t_{i+1} - \theta_{i+1} = c_2. \tag{14}$$

We solve (9) with (13) and (14) to get

$$t_i^2 K_i w(1 - w) + t_i[c_2(K_{i-1} - K_i) + 2wK_i c_1] - K_{i-1}\theta_{i-1}c_2 - K_i c_1(c_1 + t_{i+1}) = 0. \tag{15}$$

TABLE 1. Total Surface Area of a Two-Stage Countercurrent Recuperator in Relation to the Temperature between Stages (initial values $t_1 = 1000°C$; $\theta_3 = 0°C$; $w_c = 10 \cdot 10^3$ W/deg)

θ_1, °C	t_3, °C	w_h, W/deg	K_1, W/m²·deg	K_2, W/m²·deg	t_{2opt}, °C	Curve No.
700	533	$15 \cdot 10^3$	50	50	800	1
700	533	$15 \cdot 10^3$	50	100	830	2
700	533	$15 \cdot 10^3$	100	50	770	3
350	300	$5 \cdot 10^3$	50	50	580	4
350	650	$10 \cdot 10^3$	50	50	825	5

Then

$$t_i = \frac{(K_i - K_{i-1})\, c_2 - 2wK_ic_1}{2K_iw\,(1-w)} \pm \frac{\sqrt{[(K_i - K_{i-1})\, c_2 - 2wK_ic_1]^2 + 4K_iw\,(1-w)\,[K_{i-1}\theta_{i-1}c_2 + K_ic_1\,(c_1 + t_{i+1})]}}{2K_iw\,(1-w)}. \quad (16)$$

Note that the t_i have to be determined directly from (15) if w = 1.

In solving (8) we get a system of n − 1 equations of the type of (9) with n − 1 unknown temperatures; we solve these to determine the optimum temperatures, i.e., those which give minimal F.

Equation (15) allows one to determine t_2 directly for a two-stage exchanger. A numerical method was used to solve (7) with a Promin' computer and to derive $F = f(t_2)$ for a two-stage system (Fig. 2). Table 1 gives the initial data used in calculating the total surface area and the optimum temperatures as defined by (16).

The following conclusions are drawn from the results.

1. The total area as a function of the temperature of the hotter fluid has a turning point; (16) defines the temperatures of the hotter medium between the stages such as to make F minimal.

2. The total area decreases if one of the heat-transfer coefficients increases, no matter in which stage this occurs (curves 2 and 3). This also affects the optimal temperatures and hence the relationship between the surface areas F_1 and F_2. Since the early stages (reckoned along the course of the hotter medium) operate at the higher temperatures, they have to be made of more costly steels, so one should attempt to increase the heat-transfer coefficients in the first stages.

3. The closer w is to 1, the less the curvature of $F = f(t_2)$, i.e., the less the effect of t_2 on F.

We get two equations of the form of (16) in determining the optimal temperatures for a three-stage system; we substitute the known quantities (input data) to get

$$t_3 = \frac{a_1 \pm \sqrt{a_2 - a_3t_2}}{a_4}, \quad (17)$$

$$t_2 = \frac{a_5 \pm \sqrt{a_6 - a_7t_3}}{a_8}. \quad (18)$$

Then (17) defines the t_3 for which $F = f(t_2, t_3)$ has a conditional minimum, i.e., a minimum for some particular value of t_2. Similarly, (18) defines a conditional minimum for a given value of t_3, i.e., for a series of values of t_3 we get a family of $F = f(t_2)$ curves (Fig. 3).

The absolute minimum corresponds to the least value out of the $F = f(t_2)$ minimum.

Substitution of (17) into (18) gives a fourth-degree equation for the parameters at the absolute minimum; there are certain difficulties in solving this in general form, and therefore it is better to use numerical or graphical methods to solve (17) and (18).

An equation of degree greater than 5 is obtained in determining the minimum of the function $F = f(t_2, t_3 \ldots t_n)$ if the number of stages is more than 3; in general, it is literally impossible to derive a general solution to such an equation, and therefore numerical methods have to be employed.

A numerical method has been used with a computer to solve (7) for a three-stage system with the following input data: $K_1 = K_2 = K_3 = 50$ W/m²·deg, $t_1 = 1000°C$, $\theta_1 = 700°C$, $\theta_4 = 0°C$, $w_h = 15 \cdot 10^3$ W/deg, $w_c = 10 \cdot 10^3$ W/deg; Fig. 3 shows the results. Also, (17) and (18) were solved graphically for the above conditions. The temperatures corresponding to the absolute minimum were $t_2 = 870°C$, $t_3 = 710°C$.

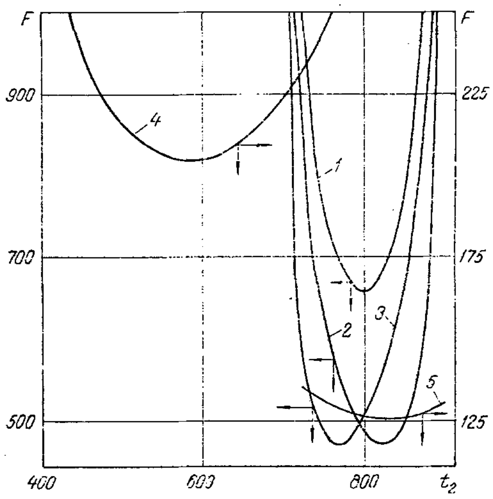

Fig. 2

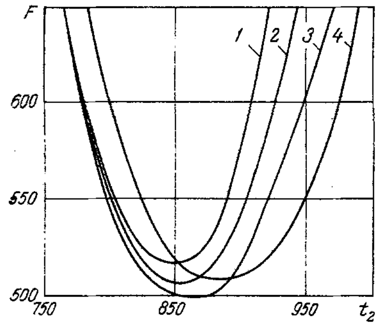

Fig. 3

Fig. 2. Total surface area F (m^2) as a function of t_2 (°C), the temperature of the heating medium between stages, for a two-stage countercurrent heat exchanger: 1-5) see Table 1.

Fig. 3. Total surface area F (m^2) as a function of t_2 (°C), the temperature of the heating medium between stages, for a three-stage countercurrent heat exchanger for t_3 (°C) of: 1) 650; 2) 670; 3) 710; 4) 770.

The computer results agree with those from graphical solution of (17) and (18).

Comparison of Fig. 2 (curve 1) and Fig. 3 (curve 3) shows that the total area decreases as the number of stages increases, other conditions being the same.

Therefore, the total area in a countercurrent heat-exchange system is dependent on the choice of the intermediate temperatures; if the temperatures between stages are chosen arbitrarily, the temperature of the cooler medium in the stage with the larger area may be less than that in the stage with the smaller area.

NOTATION

t_1, t_n, temperatures of heating fluid at the inlets to the first and last stages; θ_1, θ_n, temperatures of the heated fluid at the outlets from the first and last stages; w_h, w_c, water equivalents of the heating and heated (cold) fluids; $w = w_h/w_c$, ratio of water equivalents of the heating and heated fluids; K_1, K_i, K_n, heat-transfer coefficients of the first, i-th, and last stages; F_1, F_i, F_n, heat-transfer areas of the first, i-th, and last stages; F, total area of all the stages; a_i, constant coefficients.